Problem 1. E85 ethanol blended fuel is not required to be 85% ethanol. In fact, so-called E85 fuel at the pump can contain as low as 50% ethanol! E98 fuel can be mixed with E85 pump fuel to yield a blended true E85 fuel sutable for racing.

Given that G_{pump} gallons of pump fuel contains $X \in [.50, .85]$ percent ethanol and that E98 fuel contains exactly 98% ethanol, how many gallons of E98 fuel, G_{E98}, should be mixed to yield $G_{TrueE85}$ gallons of true E85 blend?

Notice that we have the following system of linear equations

$$\begin{cases} G_{TrueE85} = G_{pump} + G_{E98} \\ (.85)G_{TrueE85} = XG_{pump} + (.98)G_{E98}. \end{cases} \tag{1}$$

The first equation gives us the total number of gallons of true E85 fuel and the second equation gives us the ratio to achieve true E85 fuel. We solve this system by substitution. With the first equation, we have

$$(.85)G_{TrueE85} = (.85)(G_{pump} + G_{E98}) = (.85)G_{pump} + (.85)G_{E98} = XG_{pump} + (.98)G_{E98}. \tag{2}$$

Moving like terms together,

$$(.85 - X)G_{pump} = (.13)G_{E98}. \tag{3}$$

Solving for the number of gallons of E98 fuel required to ensure a proper mixture,

$$G_{E98} = \frac{.85 - X}{.13} G_{pump}. \tag{4}$$

Additionally, the total number of gallons of true E85 fuel this mixture will make is given by

$$G_{TrueE85} = G_{pump} + G_{E98} = G_{pump} + \frac{.85 - X}{.13} G_{pump} = \left(1 + \frac{.85 - X}{.13}\right) G_{pump}. \tag{5}$$

■

Example 1. If we measure 6 gallons of E85 pump gas to be 78% ethanol, how gallons of E98 should we mix into the E78 fuel to yield a true E85 mixture? How many gallons of E85 will this produce?

Since $X = .78$, we have

$$G_{E98} = \frac{.85 - X}{.13} G_{pump} = \frac{.85 - .78}{.13}(6) = \left(\frac{7}{13}\right)(6) \approx 3.23 \text{ gallons}. \tag{6}$$

This mixture will make 9.23 gallons of true E85 fuel. ■

Problem 2. If it costs p_{pump} dollars per gallon of pump E85, p_{E98} dollars per gallon of E98, and p_{E85} dollars per gallon of true E85 (ordered online), when is it cheaper to mix the fuel versus buy it?

The cost of blending fuel is $p_{pump}G_{pump} + p_{E98}G_{E98}$ where the cost of buying an equivalent amount of E85 would be $p_{E85}(G_{TrueE85})$. Substituting as above, we have the cost of blending fuel as

$$\begin{aligned} C_{blending} \doteq p_{pump}G_{pump} + p_{E98}G_{E98} &= p_{pump}G_{pump} + p_{E98}\frac{.85 - X}{.13}G_{pump} \\ &= \left(p_{pump} + p_{E98}\frac{.85 - X}{.13}\right)G_{pump}. \end{aligned} \tag{7}$$

The cost of E85 online can be expressed as a funtion of the number of gallons of pump fuel we would need to buy to equal the same volume of fuel. Indeed, by the first equation of the system,

$$C_{E85}\cdot = p_{E85}(G_{pump} + G_{E98}) = p_{E85}\left(1 + \frac{.85 - X}{.13}\right)G_{pump}. \tag{8}$$

Once we compare the two cost functions, we can cancel out the number of gallons of pump gas to directly compare prices. If

$$p_{E85}\left(1 + \frac{.85 - X}{.13}\right) \leq p_{pump} + p_{E98}\frac{.85 - X}{.13} \tag{9}$$

then you should buy the premade E85 fuel. Otherwise, if

$$p_{E85}\left(1 + \frac{.85 - X}{.13}\right) > p_{pump} + p_{E98}\frac{.85 - X}{.13} \tag{10}$$

you should blend the fuel. ■

Example 2. For the above example, assume that the price of pump E85 is 3.50 a gallon, E98 can be purchased for 4.10 a gallon, and E85 is 12.00 a gallon. Does it make more sense to blend or buy the E85?

Well, the cost of blending is

$$p_{pump} + p_{E98}\frac{.85 - X}{.13} = \left(3.5 + (4.1)\frac{.85 - .78}{.13}\right) \approx 5.71 \tag{11}$$

and the cost of buying is

$$p_{E85}\left(1 + \frac{.85 - X}{.13}\right) = (12)\left(1 + \frac{.85 - .78}{.13}\right) \approx 18.46. \tag{12}$$

Thus you should blend the fuel. Notice that these factors, when multiplied by the number of pump gas gallons, equal to the straightforward way of calculating total cost. Indeed, $(5.71)(6) = 34.243 = (3.23)(4.1) + (6)(3.5)$ and $(18.46)(6) = 110.76 = (9.23)(12)$ within a rounding error. ■